Off-Shell Rho-Omega Mixing Through Quark Loops With Non-Perturbative Meson Vertex And Quark Mass Functions

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Abstract

The momemtum dependence of the off-shell ρ-ω mixing amplitude is calculated through a two-quark loop diagram, using non-perturbative meson-quark vertex functions for the ρ and ω mesons, as well as non-perturbative quark propagators. Both these quantities are generated self-consistently through an interlinked BSE-cum-SDE approach with a 3D support for the BSE kernel with two basic constants which are pre- checked against a wide cross section of both meson and baryon spectra within a common structural framework for their respective 3D BSE's. With this pre-calibration, the on-shell strength works out at -$2.434\delta(m_q^2)$ in units of the change in "constituent mass squared", which is consistent with the e^+e^- to $\pi^+\pi^-$ data for a u-d mass difference of 4 MeV ,while the relative off-shell strength (0.99 ± 0.01) lies midway between quark-loop and QCD-SR results. We also calculate the photon-mediated ρ-ω propagator whose off-shell structure has an additional pole at q^2=0. The implications of these results vis-a-vis related investigations are discussed.

During the last couple of years, the problem of charge symmetry violation (CSV) in nuclear physics [1], especially a possible resolution of the so-called Nolen-Schiffer anomaly (NSA) [2] by the $\rho_0 - \omega$ mixing contribution to the N-N force [3],has come in for a good deal of attention [4]-[8]. The renewed interest was triggered by the possibility of an off-shell momentum dependence [4] in the $\rho - \omega$ mixing amplitude which would be expected to arise through any reasonable form of dynamics generated by an intermediate fermion-antifermion loop,wheher at the quark level (u-d mass difference) [4] or at the nucleon level (n-p mass difference) [7]. Since the original suggestion [4], several more quark level calculations [5, 6, 8] have appeared for a quantitative handle on the

issue of momentum dependence of the $\rho - \omega$ [6, 8] and the analogous $\pi - \eta$ [5] amplitudes, using respectively the results of the effective chiral Lagrangians [9, 10], the field- theoretic language of Schwinger-Dyson Equation for the quark propagator [11], and the methodology [12, 13] of QCD sum rules [13]. All these calculations reveal in varying degrees the effect of off-shell momentum dependence of the $\rho - \omega$ amplitude on the physics of the N-N potential in its sensitive region. The estimates however seem to vary over a wide range, from relatively gentle variations via the quark loop [4, 6] or nucleon loop [7] mechanisms, to rather large effects [8] through the QCD sum rule method, though none of these methods seem to be free from uncertainties of input parametrizations. The results are expressed by a certain output parameter λ which is roughly a measure of the degree of off-shellness of the $\rho - \omega$ mixing amplitude $\theta(q^2)$ defined on a linear scale through its dependence on the variable q^2 [5, 8] (but adapted to the euclidean metric) as follows :

$$\theta(q^2) = \theta(M^2)[1 - \lambda(1 + (q^2/M^2)]\tag{1}$$

where M is the mean $\rho - \omega$ mass and the magnitude of λ is the quantity of main dynamical interest [4, 5, 6, 7, 8].

1 Quark-Loop Method in 4D vs. 3D BSE

In the QCD-SR method, the inputs are $< q\bar{q} >$, $\delta < q\bar{q} >$ and the current u, d masses which are more or less under control. However the uncertainties arise from the 'matching' of the two sides of the 'duality' equation by the 'standard' methods [12, 13] to obtain the necessary stability in the sum-rule structure, which is the key to the determination of the hadronic parameter λ [8].

In the quark-loop method [4, 6], on the other hand, the dynamics resides in the hadron-quark vertex function and constituent quark mass [4], or better, the dynamical mass function [6], and these two quantities in turn are inter-linked through the Schwinger-Dyson and Bethe-Salpeter Equations, a formalism which, though well known since the seventies [11],has been greatly revived in the nineties [14, 15]. These approaches also require in practice a generous degree of parametrization [14, 15] of the basic entities, since any exact solution of these coupled equations is still a distant dream. Therefore any attempt to adapt this methodology to the present problem [6] must also share the corresponding parametric uncertainties [15], without prior checks from other sectors of hadron physics, notably spectroscopy as well as some sensitive coupling constants and form factors. A broad perspective on these approaches [14, 15] has been discussed more fully elsewhere [16],in the context of an alternative program with a similar philosophy [17]-[20] whose continued emphasis on the spectroscopy sector stems from its sensitivity to the 'gluon-like prop-

agator' in the infrared region' (a paraphrase for the '$q - \bar{q}$ potential' in the more ordinary language).

We shall not go into the relative merits of the QCD-SR [12] and the BSE-cum-SDE methods, which have been discussed elsewhere [16]. However in the context of the latter it will be useful to make a distinction between two broad types, the 'spectroscopy-oriented' type [16] $\rightarrow$ [20] which depends on a basically 3D approach, and the more "orthodox" 4D type [14, 15]. For purposes of this paper we shall term these two BSE-cum-SDE methods as "3D" and "4D" BSE's respectively for short. Both make use of the two-quark loop integration to calculate the rho-omega amplitude but this difference in terminology emphasises the difference in the parametrizations of the infrared region of the gluonic propagator which are 3D and 4D respectively. While the 4D form is prima facie more natural, the theoretical reasons for the 3D form are no less persuasive and the interested reader will find the necessary details on its theoretical motivations [22] from various angles in [16, 20]. Here we shall merely cite the chief expreimental reason, viz., the $O(3)$-like spectra in the PDG-tables [23] continually for four decades which provides the bed-rock of foundation for any theoretical effort at a microscopic description of quark structure, and our 3D BS-programme [16]-[21] has been specifically designed to meet this requirement.On the other hand, a literal consequence of the 4D form of parametrization of the infrared part (confining) of the gluon propagator [15] would be to predict $O(4)$-like spectra which contradicts experiment [16]. The reason why such $O(4)$-like results are not entirely visible in some of these spectral predictions [15] is merely because of their consideration of mainly the ground state masses $L = 0$, since the fuller implications of the 4D forms would not start showing up until the predictions include the L-excited states [16].

Since some quark loop results of the 4D-BSE [15] are already available [6], along with those of the QCD-SR analysis [8], it should be of considerable interest, even without prejudice to the question of $O(3)$ versus $O(4)$-like spectra (important as it may be in its own right), to record for comparison the corresponding results of the 3D-BSE [16], in view of their parameter-free nature. This may be useful in the context of the current controversy [4, 6, 7, 8] on the off-shell strength of the $\rho - \omega$ mixing amplitude which is an important parameter for charge symmetry breaking in the $(N - N)$ force. We recall in this connection that the 3D-BSE formalism is specifically calibrated to both $q - \bar{q}$ [18] and qqq [19] spectra, both in excellent accord with data [23], as well as to a representative list of hadron couplings [24, 16, 17]. All this has been obtained with just two basic constants C_0, ω_0 *common* to both types, since a third input, the quark mass (constituent), gets dynamically generated thruogh the (chiral-symmetry breaking) solution of the SDE [11, 16], so that in this (spectroscopy-oriented) BSE-cum-SDE approach there is practically no scope for any free parametrization beyond the ones noted above, a condition which

is probably important for the determination of the rather sensitive parameters $\theta(M^2)$ and λ under study.

2   3D BSE-cum-SDE Formalism

The purpose of this paper is to present the results of this calculation in the most economical fashion, omitting all but the essential details. To that end we shall use [8] for the definitions and notations of the crucial parameters involved, and calibrate our language to that of [4, 6] as far as possible for the definition of the loop integrals, except for the implicit understanding of an euclidean metric notation underlying our formulation, and the use of P for their q notation since the latter has in all our formulations [16, 17, 20] stood for the internal 4-momentum of the quarks within the hadron, while P is the 4-momentum of the (composite) hadron. As to the 3D-BSE formalism itself we shall make free use of [16, 17], but recapitulate some essential results so as to keep the paper within reasonably self-contained limits.

The quantities we shall explicitly calculate in our formalism are : (i) the function $\theta(P^2)$ with an explicit proportionality to $\delta(m_q^2)$ where m_q^2 is the constituent (dynamical) quark mass 'squared', which is obtained directly from an analytical formula (given below) for $\Pi(P^2)$ [8], by a simple process of differentiation w.r.t. m_q^2 ; (ii) the parameter λ [8] which can be explicitly identified from the linear dependence of this quantity on the inverse meson propagator $(P^2 + M^2)$; (iii) the analogous $\rho - \omega$ potential mediated by an intermediate photon, so that its full off-shell structure is a chain of two linear off-shell quantities $g_{\rho-\gamma}$ and $g_{\omega-\gamma}$ to be compared with the θ-function ,eq,(1), for $\rho - \omega$ mixing due to the "strong" effect of u, d difference which involves this linear factor in the off-shell quantity $(P^2 + M^2)$ only once. (Despite the comparative weakness of the e.m. effect, its off-shell scenario is, on this account, somewhat different from that of the (strong) $u - d$ effect; this point is discussed further at the end).

As to the actual numerical values, the only unknown quantity in our formalism is the $u - d$ mass difference which we shall keep as a free multiplicative factor for $\theta(M^2)$, to facilitate discussion at the end on this point. The main results are:

$$\theta(M^2) = -1289(MeV) \cdot \delta(m_q); \lambda = 0.99 \pm 0.01 \qquad (2)$$

where for $\delta(m_q)$ it is usual [6] to take $m_d - m_u$.

Our formalism is based on the Covariant Instaneity Ansatz (CIA) [17] which gives the Bethe-Salpeter Kernel $K(q, q')$ for a quark-antiquark system a 3D support expressed through its dependence on the component of q_μ trans-

verse to P_μ, for which we use a hat notation,i.e.,

$$\hat{q}_\mu = q_\mu - q \cdot PP_\mu/P^2 \tag{3}$$

so that $K(q, q') = K(\hat{q}, \hat{q}')$ for a 3D support [17]. As a result of this ansatz, there is an exact interconnection between the 3D and 4D forms of the BSE [17] and the hadron-quark vertex function $\Gamma_H(q, P)$ becomes a function $\Gamma_H(\hat{q})$ of a single argument $\hat{q}_\mu$. It is usually convenient to take out the Dirac matrix from this structure, viz.,γ_5 for a pion, $i\gamma \cdot \rho$ for a rho-meson, etc.; the multiplying scalar factor $\Gamma_h(\hat{q})$ which carries the dynamical information has the following universal structure [17] :

$$\Gamma_h(\hat{q}) = N_H \times D(\hat{q}) \otimes \phi(\hat{q})/(2\pi)^{5/2} \tag{4}$$

Here $D(\hat{q})$ is a 3D denominator function, and $\phi(\hat{q})$ the corresponding wave function which together satisfy a Lorentz-covariant Schroedinger-like equation of the form $D\phi = \int K\phi$, representing the 3D reduction of the 4D BSE as a result of the above ansatz. The quantity N_H is the standard 4D BS normalizer which goes with the vertex function (4). The D-function for equal mass kinematics has the simple form

$$D(\hat{q}) = 4\omega(\omega^2 - M^2/4); \omega^2 = m_q^2 + \hat{q}^2, \tag{5}$$

while the ϕ-function is model dependent. In particular, a gaussian form

$$\phi(\hat{q}) = exp(-\hat{q}^2/(2\beta^2)) \tag{6}$$

emerges (for harmonic confinement) as a solution of the 3D BSE, with β^2 obtained *analytically* from the input structure of the BS-kernel [16, 18] and checked against Spectroscopy [23]. Its value for the $\rho - \omega$ case is 0.0692 [18]. In eq.(5), M is the hadron mass and m_q the constituent (dynamical) quark mass. Its momentum dependence was obtained in [16] by simply relating the quark mass function to the pion-vertex function which must reduce to each other in the chiral limit of vanishing pion mass $(M_\pi = 0)$, by virtue of the Ward-Takahashi identity for the axial-vector vertex function [11]. Therefore by specializing eqs.(4-6) to the pion case in the limit $M_\pi = 0$, one immediately obtains the formula [16] :

$$m(\hat{p}) = m_q^{-2}(m_q^2 + \hat{p}^2)^{3/2} \cdot exp(-\hat{p}^2/2\beta_\pi^2) \tag{7}$$

where the quantity β_π^2 (=0.031 for the pion case [18]) is still governed by the same BS-dynamics [16, 18], but now (because of the goldstone nature of the pion in the chiral limit) the normalization has had to be fixed anew by identifying the "constituent" mass m_q with this function at its zero-momentum

limit $(m(0) = m_q)$. In terms of $m(\hat{p})$ the non-perturbative quark propagator $S_F(p)$ is now given by

$$(S_F(p))^{-1} = i(m(\hat{p}) + i\gamma \cdot p) \tag{8}$$

where the Landau gauge is understood $(A(p^2) = 1, [14, 16])$ and $m(\hat{p})$ is given by (7). This non-perturbative mass function was employed in [16] for evaluating the quark condensate $< q\bar{q} >$ as an explicit quadrature :

$$< q\bar{q} >= \frac{6}{\pi^2} \int d^3\hat{p} \frac{m(\hat{p})}{\sqrt{\hat{p}^2 + m(\hat{p})^2}}$$

giving a value in the QCD-SR range [12]; the meaning of $\hat{p}_\mu$ was also clarified therein.

An important property of the structure (5) for the quantity $D(\hat{q})$ in the hadron-quark vertex function is that it prevents, through a general cancellation mechanism [17, 20], the occurrence of overlapping pole effects due to integration over the time-like component of the loop-momentum in any quark-loop integral, and thus automatically pre-empts the possibility of any "free" propagation of quarks that might otherwise occur. It thus may be regarded as a simple 3D alternative to the construction of quark propagators as entire functions through more elaborate models [15, 6], but with the added benefit of a parameterfree description (c.f.,[6]). This structure will play a key role in simplifying the loop-integral for the meson self energy operator from its 4D scalar form, eq.(10), to the 3D form, eq.(12), as given below.

3 The $\rho \to \omega$ Amplitude

After collecting these essential ingredients of the "3D" BSE formalism, we now turn to the central quantity, viz.,the two-loop contribution to the meson self-energy operator $\Pi_{\mu\nu}(P^2)$ [4, 6, 8] which is expressible as

$$\rho_\mu \Pi_{\mu\nu}(P^2)\omega_\nu = i(2\pi)^4 \times \int d^4q \Gamma_h(\hat{q})^2 Tr[i\gamma \cdot \rho S_F(q + P/2)i\gamma \cdot \omega S_F(-q + P/2)] \tag{9}$$

where $\Gamma_h(\hat{q})$ is the scalar part of the vertex function defined by eq.(4), and the ρ,ω symbols on both sides of (9) stand for their respective polarization vectors. At this stage the scalar vertex function is common to ρ and ω, since their mass difference due to the $u - d$ effect will be automatically taken care of via standard differentiation w.r.t. m_q^2, c.f.[4];(see below). Simplifying the trace in eq.(9) and checking on current conservation (which is routinely satisfied) we can write $\Pi_{\mu\nu}(P^2)$ as $(\delta_{\mu\nu} - P_\mu P_\nu \cdot P^{-2})\Pi(P^2)$ where, following any one of [17, 20, 25],

$$\Pi(P^2) = 2i(2\pi)^4 N_V^2 \int d^4q[D^2(\hat{q})\phi^2(\hat{q}) \cdot (\Delta_1 + \Delta_2 - P^2 - 4\hat{q}^2/3)/(\Delta_1\Delta_2)] \tag{10}$$

where [17, 20, 25]

$$\Delta_{1,2} = m_q^2 + (P/2 \pm q)^2; (P^2 = -M^2) \qquad (11)$$

The integration over the longitudinal (time-like) component of q_μ, viz., $Md\sigma$, (σ equals $q \cdot P/P^2$), is carried out again as in [17, 20, 25] wherein the structure (5) of the D function ensures an exact cancellation of the effects of overlapping singularities arising from the σ-pole residues. The resultant 3D integration over $d^3\hat{q}$ is expressible as

$$\Pi(P^2) = -2N_V^2 \int d^3\hat{q}\phi^2(\hat{q})[D^2(\hat{q})/\omega - D(\hat{q}) \cdot (P^2 + 4\hat{q}^2/3)]. \qquad (12)$$

Eq.(12) brings out explicitly, without further ado, the linear structure of the mass operator in the off-shell variable P^2. The BS normalizer N_V^2 in eq.(12) is itself an integral of the same kind as $\Pi(P^2)$, and is formally defined for any V-meson through the equation [24, 17]:

$$2iP\mu N_V^{-2} = (2\pi)^4 \int d^4q\Gamma_h(\hat{q})^2 Tr[i\gamma \cdot V S_F(q+P/2)i\gamma_\mu S_F(q+P/2)i\gamma \cdot V S_F(-q+P/2)] \qquad (13)$$

whose integral over the time-like component of $q\mu$ can be carried out exactly as above to give a formula analogous to eq.(12) :

$$N_V^{-2} = 2 \int d^3\hat{q}\phi^2(\hat{q})4\omega \times [\omega^2 - \hat{q}^2/3] = 0.0502(GeV)^{-6} \qquad (14)$$

The quantities ω and D in both equations (12,13) are defined as in eq.(5) which in turn carries the explicit information on the m_q^2 dependence of both these quantities. This fact facilitates a simple differentiation w.r.t. m_q^2 under the integral signs in eqs.(12,13) in order to evaluate $\delta[\Pi(P^2)]$ which precisely represents, with no further normalization, the desired quantity $\theta(P^2)$ defined in eq.(1), while the values of its two crucial parameters as predicted by this model are already listed in eq.(2). In obtaining the latter we have used the equality of $\delta(m_q^2)$ with $2m_q \cdot \delta(m_q)$, and employed the 'spectroscopic' value 265 Mev for m_q, the constituent mass [18, 19]. For the evaluation of the integrals (12,13) we have not explicitly considered the momentum variation (7) of the dynamical mass, but left it at its 'constituent' value m_q corresponding to zero momentum. This has been done mainly for simplicity and transparency in carrying out the differentiation process. Though not strictly valid. the scope of error on this account is likely to be small for two reasons : (i) the main burden of momentum variations in the two integrals (12,13) is carried by the meson-quark vertex function whose effect has been fully incorporated via eqs.(4-6); (ii) the mass function, eq(7), maintains a sort of plateau (250-300 Mev) in the region of integration which provides the bulk contributions to the integrals. Our estimate of error, based on some trial runs with the

momentum-dependent mass function, is about 10%. On the other hand the explicit analytic structures in m_q^2 of the integrals (12,13) greatly minimize the possibility of further numerical errors that would be inherent in the differentiation process in the absence of a ($non-perturbative$) analytical form which is usually more difficult to ensure than, e.g.,a point vertex structure [4], without additional parametric assumptions on the way, e.g., [6].

Before comparing our results with others, we wish to record for completeness the predictions of this model on the photon-mediated chain of $\rho - \gamma - \omega$ mixing amplitude which we denote by $\theta_\gamma(P^2)$ in the same relative normalization as eq.(1). Here we need no longer distinguish between m_u and m_d and take a simple proportionality of the $\rho - \gamma$ and $\omega - \gamma$ amplitudes to a common dynamical quantity $g_V(P^2)$ defined by

$$g_V(P^2)V_\mu = -i \int d^4q \Gamma_h(\hat{q}) \times Tr[i\gamma \cdot V S_F(q+P/2)i\gamma_\mu S_F(-q+P/2)]/\sqrt{2}, \quad (15)$$

the multiplicity factors being e and $e/3$ respectively, and V_μ standing collectively for ρ or ω. The other symbols are as defined in eq.(9) and earlier. The evaluation of $g_V(P^2)$ is on lines similar to eq.(9),but actually simpler, and leads to the explicit formula

$$g_V(P^2) = 4\sqrt{(3/2)}\beta^3 N_V[2m_q^2 + 4\beta^2 - (P^2 + M^2)/2] \quad (16)$$

Writing it in a forn analogous to eq.(1), we have

$$g_V(P^2) = f_V(M^2)[1 - \mu(1 + P^2/M^2)]; \quad (17)$$

where the on-shell value $f_V(M^2)$ and the off-shell coefficient μ are

$$g_V(M^2) = 0.1608(Gev)^2; \mu = M^2/[4(m_q^2 + 2\beta^2)] = 0.7197. \quad (18)$$

The final result for the complete photon-mediated ρ-ω amplitude is

$$\theta_\gamma(P^2) = \frac{e^2}{3}g_V(P^2)\frac{1}{-P^2}g_V(P^2), \quad (19)$$

where we have explicitly shown the photon propagator in the middle, to bring out the 'extended' nature of the off-shell extrapolation due to the photon-mediated mixing compared to that due to the $u - d$ effect, despite the smallness of (19) compared to (1). Unlike (1), there is no uncertainty in (19) within this model, though the on-shell value ($P^2 = -M^2$) is a bit too high (see discussion below).

$$\theta_\gamma(M^2) = +1316(Mev)^2 \quad (20)$$

The off-shell effect, on the other hand, is best expressed through the corresponding N-N potentials [8] which are given by

$$V(\rho - \omega) = -[\theta(M^2)/2M][1 - (2\lambda/Mr)]exp(-Mr); \quad (21)$$

$$V(\rho-\gamma-\omega) = [\theta_\gamma(M^2)^2/M^4]\cdot[(i-\mu)^2/r+[(2\mu-1)/r]exp(-Mr)-(M/2)exp(-Mr)] \tag{22}$$

respectively, where a common factor $g_{\rho N}$ $g_{\omega N}$ [8] has been suppressed from the last two equations. Eq.(21) has no counterpart of the $1/r$ term in (22).

4   Results and Discussion

To put the results of this investigation in perspective with those in [4]→[8] we should first note that in this spectroscopy-rooted approach there is little scope for any significant variation of the input parameters (ω_0, C_0, and m_q) whose respective values (158 Mev, 0.27, and 265 Mev) can be traced all the way back to the BS-Kernel itself [18], without effecting a simultaneous change in the (already good) fits [18] to the observed meson spectra [23], and in the more recent (equally good) fits [19] to the baryon spectra [23] with these very parameters. It is with this constraint that the numbers obtained above may be viewed vis-a-vis those in [4]→[8], especially in respect of the off-shell parameter λ, eq(1), which can be compared with almost all of them. However, the on-shell value $\theta(M^2)$,eq(2), is quite specific in this model, and could at at best be compared with the predictions of, say, chiral Lagrangian models [9, 10], except that the available prediction [5] refers to π^0-η mixing and cannot be used for a direct comparison.

The only uncertainty in our on-shell value, eq.(2), arises from a corresponding uncertainty in the value of $\delta(m_q)$ for which a natural substitute, *ala* Politzer [26], would be $(m_d - m_u)$. The latter quantity has been discussed in great detail in [8] to which we refer the interested reader, but for a definitive estimate it should be reasonable to take a value, say, 4 Mev [6] which is well within the limits of the [8] analysis. With this value we get $\theta(M^2)$ equal to $(-5156)MeV^2$ which should be compared with the value (-4520 ± 600) obtained from $e^+e^- \rightarrow \pi^+\pi^-$ data [27], after taking account of the $\rho - \gamma - \omega$ chain which gives a smaller contribution of opposite sign, viz., +1316, eq.(20). Its inclusion gives the net value -3840 which is still within the experimental range [27], (taking account of the uncertainties of the u-d mass difference).

The somewhat larger value of $\theta_\gamma(M^2)$ compared to the "VMD" value [28] 610 MeV^2 quoted in [8] may in turn be related to the quantity $g_V(M^2)$,eq.(18), which gives 0.1608 GeV^2. This number, when divided by $M_\rho = 0.775$ GeV, precisely translates,in the QCD-SR [29] notation, to the result $f_\rho = 215$ MeV, to be compared to the quoted value of 200 MeV [29] needed for agreement with the $\rho\rightarrow e^+e^-$ width. This is the extent of our overestimate of $\theta_\gamma(M^2)$ compared to the VMD-value [28, 8], but nevertheless tolerable enough to warrant a discussion (below) of the off-shell aspects of ρ-γ-ω mixing along with that of the main (u-d) term.

The off-shell prediction is dominated by the parameter λ, eq.(1) at the

value (0.99), eq.(2), and its photonic counterpart μ defined in eq.(18) at the value (0.720). Our value of λ is rather below the QCD-SR range (1.43-1.85) [8], implying a "softer" off-shell effect in this quark-loop model than the "harder" effect in the QCD-SR approach, as already noted in [8] for QCD-SR versus quark-loop methods: A smaller value of λ would tend to postpone the onset of attenuation of the $\rho - \omega$ mixing potential due to the off-shell effects, to somewhat shorter distances, as measured by the "critical distance" [8] r_c = $2\lambda_M$, which is also seen from eq.(21). In a similar way, the off-shell effect of the photon-mediated $\rho - \omega$ mixing, as measured by the parameter $\mu =$ 0.720,eq.(18), produces the potential,eq.(22), but its $(1/r)$-term has no counterpart in eq.(21). Taking note of the *opposite* signs of the two effects, the following scenario emerges: The two short range terms of (21) get duly reduced by the two corresponding terms of (22) by about 20-25 %. However, the long range $(1/r)$-term of (22), which has no counterpart in (21), reinforces the $exp(-Mr)$ term of the latter, again by about 20-25% near the critical distance [6, 8], but continues with increasing strength down to shorter distances and therefore further postpones the attenuation by another (small) amount. For brevity we omit further discussion [8].

Finally we wish to comment on the magnitude of our λ-value, 0.99 ± 0.01 vis-a vis other determinations [4, 5, 6, 7, 8].

We have checked on the possible variation in this quantity due to the (neglected) effect of the momentum dependence of the dynamical mass eq.(7), and found the effect to be $\leq 10\%$. There is little scope for further variation in this otherwise 'rigid' description, unless a totally different set of input parameters (C_0, ω_0,m_q) from the ones [18, 19] employed here, produces an equally good fit [18, 19] to the observed spectra [23], which is rather unlikely. Nevertheless this value seems to lie about midway between other quark-loop calculations [4, 5, 6] and QCD-SR results [8], though somewhat nearer to the former than to the latter; rather surprisingly, it is quite close to the nucleon-loop value [7] of about unity [8]. It is also in fair agreement with the corresponding results [5] for π^0-η mixing obtained from chiral Lagrangian models [9, 10], though a similar result for the analogous case of ρ- ω mixing by the same method [9, 10] is not yet available. Of course a $non - linear$ dependence of $\theta(P^2)$ on P^2, such as attempted in [6], may well change this (linear) scenario, but this requires more effort.

To summarise, we have outlined an explicit calculation of the ρ- ω mixing amplitude, both on- and off- shell, in the form expressed by eqs.(1-2), using a 3D BSE-cum-SDE approach which is attuned to hadron spectroscopy of both varieties simultaneously [18, 19]. The on-shell value agrees with experiment [27], while the off-shell parameter λ is rather close to unity, signifying a change of sign for $\theta(q^2)$ in just the transition region between space-like and time-like momenta.

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